OCR
Oxford Cambridge and RSA

Wednesday 24 June 2015 – Morning

A2 GCE MATHEMATICS

4734/01 Probability & Statistics 3

QUESTION PAPER

3150485028

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4734/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

© OCR 2015 [J/102/2706]
DC (AC) 81548/4

OCR is an exempt Charity

1 A laminate consists of 4 layers of material C and 3 layers of material D. The thickness of a layer of material C has a normal distribution with mean 1 mm and standard deviation 0.1 mm, and the thickness of a layer of material D has a normal distribution with mean 8 mm and standard deviation 0.2 mm. The layers are independent of one another.

(i) Find the mean and variance of the total thickness of the laminate. **[3]**

(ii) What total thickness is exceeded by 1% of the laminates? **[3]**

2 In a poll of people aged 18–21, 46 out of 200 randomly chosen university students agreed with a proposition. 51 out of 300 randomly chosen others who were not university students agreed with it. Test, at the 5% significance level, whether the proportion of university students who agree with the proposition differs from the proportion of those who are not university students. **[7]**

3 A tutor gave an assessment to 6 randomly chosen new eleven-year-old students. After each student had received 30 hours of tuition, they were given a second assessment. The scores are shown in the table.

Student	*A*	*B*	*C*	*D*	*E*	*F*
1st assessment	124	121	111	113	118	119
2nd assessment	127	119	114	110	120	122

(i) Show that, at the 5% significance level, there is insufficient evidence that students' scores are higher, on average, after tuition than before tuition. State a necessary assumption. **[8]**

(ii) Disappointed by this result, the tutor looked again at the first assessment. She discovered that the first assessment was too easy, in fact being a test for ten-year-olds, not eleven-year-olds. She decided to reduce each score for the first assessment by a constant integer k. Find the least value of k for which there is evidence at the 5% significance level that the students' scores have, on average, improved. **[4]**

4 A set of bathroom scales is known to operate with an error which is normally distributed. One morning a man weighs himself 4 times. The 4 values for his mass, in kg, which can be considered to be a random sample are as follows.

62.6 62.8 62.1 62.5

(i) Find a 95% confidence interval for his mass. Give the end-points of the interval correct to 3 decimal places. **[5]**

(ii) Based on these results, a y% confidence interval has width 0.482. Find y. **[4]**

© OCR 2015

5 Two guesthouses, the Albion and the Blighty, have 8 and 6 rooms respectively. The demand for rooms at the Albion has a Poisson distribution with mean 6.5 and the demand for rooms at the Blighty has an independent Poisson distribution with mean 5.5. The owners have agreed that if their guesthouse is full, they will re-direct guests to the other.

(i) Find the probability that, on any particular night, the two guesthouses together do not have enough rooms to meet demand. **[3]**

(ii) The Albion charges £60 per room per night, and the Blighty £80. Find the probability, that on a particular night, the total income of the two guesthouses is exactly £400. **[4]**

(iii) If A is the number of rooms demanded at the Albion each night, and B the number of rooms demanded at the Blighty each night, find the mean and variance of the variable $C = 60A + 80B$. State whether C has a Poisson distribution, giving a reason for your answer. **[4]**

6 In each of 38 randomly selected weeks of the English Premier Football League there were 10 matches. Table 1 summarises the number of home wins in 10 matches, X, and the corresponding number of weeks.

Number of home wins	0	1	2	3	4	5	6	7	8	9	10
Number of weeks	0	1	2	8	8	9	7	1	2	0	0

Table 1

A researcher investigates whether X can be modelled by the distribution B$(10, p)$. He calculates the expected frequencies using a value of p obtained from the sample mean.

(i) Show that $p = 0.45$. **[3]**

Table 2 shows the observed and expected number of weeks.

Number of home wins	0	1	2	3	4	5	6	7	8	9	10	Totals
Observed number of weeks	0	1	2	8	8	9	7	1	2	0	0	38
Expected number of weeks	0.096	0.788	2.899	6.326	9.058	8.893	6.064	2.835	0.870	0.158	0.013	38

Table 2

(ii) Show how the value of 2.835 for 7 home wins is obtained. **[3]**

The researcher carries out a test, at the 5% significance level, of whether the distribution B$(10, p)$ fits the data.

(iii) Explain why it is necessary to combine classes. **[1]**

(iv) Carry out the test. **[6]**

Question 7 begins on page 4.

7 A continuous random variable X has probability density function

$$f(x) = \begin{cases} kx & 0 \leqslant x < 2, \\ \dfrac{k(4-x)^2}{2} & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{10}$. **[3]**

(ii) Find E(X). **[4]**

(iii) Find the cumulative distribution function of X. **[4]**

(iv) Find the upper quartile of X, correct to 3 significant figures. **[3]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

© OCR 2015

Question			Answer	Marks	Guidance	
1	(i)		28 $4 \times 0.1^2 + 3 \times 0.2^2$ 0.16	B1 M1 A1 **[3]**	Not 4^2, 3^2	
	(ii)		$\frac{x - "28"}{\sqrt{"0.16"}}$ = 2.326 28.9	M1 B1 A1 **[3]**		
2			$H_0: p_u = p_n$ $H_1: p_u \neq p_n$ $\hat{p} = \frac{97}{500}$ =0.194 TS = $\frac{0.23 - 0.17}{\sqrt{0.194 \times 0.806 \times (\frac{1}{200} + \frac{1}{300})}}$ 1.66 "1.66" < 1.96, accept H_0 Insufficient evidence to suggest that there is a difference between the proportions.	B1 B1 M1 A1 A1 M1 A1 **[7]**	If words, must have 'population' Allow 1 error in var formula for M1 Correct var (= 0.0361) ft TS; 0.0965>5% or 0.0483>2.5% Not over-assertive. CWO (accept from 1.63)	SC1 1-tail B0B1M1A1A1M1(>1.645 rej)A0 SC2 not pooled sd = $\sqrt{(\frac{46 \times 154}{200^3} + \frac{51 \times 249}{300^3})}$ TS = 1.63 B1B0M1A1ftA0M1A1ft0.0516/7>2.5 % SC3 (both errors) B0B0M1A1ftA0M1(<1.645 acc)A0
3	(i)		$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$ $\bar{d}$ =1 Variance = $\frac{6}{5}(\frac{44}{6} - 1^2)$ = 7.6 $\frac{"1"}{\sqrt{\frac{"7.6"}{6}}}$ = 0.889	B1 B1 B1 M1A1	Allow d=0 d>0 μ_D=0 etc. If words, must have population. Allow $\bar{d}$ = -1 and consistent following work. Or sd=2.757 Allow M1 if $\frac{6}{5}$ omitted. Must have /6.	NOT μ, NOT $\bar{d}$ NOT PAIRED allow B1,(hyps) B1 (118⅔-117⅔=1) B0 M1 for var based on both assessments including /6 A0 B1 CV 2.015 or 1.812 M1 TS<CV B0 (assumption)

Question			Answer	Marks	Guidance	
			CV = 2.015 "0.889"<"2.015", do not reject H_0 AG Differences are normally distributed.	B1 M1 B1 **[8]**	Or p=0.2074/5 (>5%) Must be TS < CV (AG)	CV must be from t, not z.
	(ii)		New $\bar{d} = 1 + k$ Var/sd unchanged $\frac{"1"+k}{\sqrt{\frac{"7.6"}{6}}} >$ "2.015" $k > 1.27$ i.e. $k = 2$	B1 B1ft M1 A1 **[4]**	Must be same values as **(i)**. Allow =, if inequality soi later. Must have /"6". NOT $k = 1$.	SC If B1B1M0, allow B1 for 1.27 seen.
4	**(i)**		$\bar{x} = 62.5$ $\hat{\sigma} = 0.294(392)$ $t = 3.182$ "62.5" + "3.182"× "0.294"/2 [62.032, 62.968]	B1 B1 B1 M1 A1 **[5]**	or var=13/150 =0.0866... any t or z=1.96. $\bar{x} \pm \frac{t\sigma}{2}$ Allow 4 or more dp.	Allow 0.087 for this mark.
	(ii)		"0.294" $t/2$ oe = 0.241 soi 1.637 80% CAO	B1ft B1 B1 B1 **[4]**	Must be t. z=B0. If t written, but z used B0 Allow 1.63,1.638,1.64	
5	**(i)**		$A + B \sim$ Po(12) seen 1 – 0.7720 0.228	B1 M1 A1 **[3]**		

Question			Answer	Marks	Guidance	
	(ii)		A = 0 & B = 5 AND A = 4 & B = 2 identified	B1	These 2 pairs only. Allow B=5 alone (+A4,B2) for this mark.	
			$e^{-6.5} \times e^{-5.5}\frac{5.5^5}{5!} + e^{-6.5} \times \frac{6.5^4}{4!} \times e^{-5.5} \times \frac{5.5^2}{2!}$	B1;B1	Each product seen (not nec added), or 0.0015x0.1714;0.11182x0.06181	Or 0.000258;0.00691. Allow from tables. Eg 0.5289-0.3575=0.1714
			0.00717	B1 **[4]**		
	(iii)		Mean = 830	B1		
			Var = $60^2 \times 6.5 + 80^2 \times 5.5$	M1		
			= 58 600	A1		
			Var ≠ Mean , so no.	B1ft **[4]**	Any correct reason	eg Not all integer values possible.
6	(i)		$171 \div 38$	M1	Attempt at Σ f*x*/*38*	Allow divisions in either order, or combined.
			$\div 10$	M1	Use μ=np	
			0.45 AG	A1 **[3]**	CWO	
	(ii)		$^{10}C_7 \times 0.45^7 \times (1-0.45)^3$	M1	=0.0746	Or tables. 0.9726-0.8980
			$\times 38$	M1		
			2.835 AG	A1 **[3]**		
	(iii)		Some Es < 5	B1 **[1]**	Must have 'expected'	
	(iv)		HW 0-3 | 4 | 5 | 6-10 Obs 11 | 8 | 9 | 10 Exp 10.109 | 9.058 | 8.893 | 9.940	B1	May be implied.	
			$\frac{(11-"10.109")^2}{"10.109"} + \ldots..$	M1		
			0.204	A1	Allow 0.2	

Question			Answer	Marks	Guidance	
			CV = 5.991 TS < CV, do not reject NH There is insufficient evidence that the data is not B(10, 0.45)	B1 ft M1 A1 **[6]**	ft correct CV for df = classes – 2 Allow if incorrect CV from correct tail.	B0M1A0B1ftM1A0 max for n(classes)≠4.
7	**(i)**		$\int_0^2 kx\mathrm{d}x + \int_2^4 \frac{k}{2}(4-x)^2\mathrm{d}x = 1$ $2k + \frac{8k}{6}$ (= 1) $k = \frac{3}{10}$ AG	M1 B1 A1 **[3]**	Ignore limits for this mark.	
	(ii)		$\int_0^2 kx^2\mathrm{d}x + \int_2^4 \frac{kx}{2}(4-x)^2\mathrm{d}x$ $\frac{1}{10}x^3, \frac{3}{20}(8x^2 - \frac{8x^3}{3} + \frac{x^4}{4})$ 1.8	M1 B1,B1 A1 **[4]**	$\frac{-x(4-x)^3}{3} - \frac{(4-x)^4}{12}$ *or* $\frac{x^2(4-x)^2}{2} + \frac{4x^3}{3} - \frac{x^4x^4}{44}$ from int by parts. Ignore limits for these marks.	
	(iii)		$\frac{3x^2}{20}$ $(0 \leqq x < 2)$ "0.6"+ $\frac{k}{2}\int_2^x (4-x)^2\mathrm{d}x$ oe $1 - \frac{(4-x)^3}{20}$ oe, $(2 \leq x \leq 4)$ 0 $x < 0$, 1 $x > 4$	B1 M1 A1 B1 **[4]**	Or $-\frac{(4-x)^3}{20} + c$ and attempt to find c. $0.05x^3-0.6x^2+2.4x-2.2$ 0, 1 and all ranges.	

Question			Answer	Marks	Guidance	
	(iv)		$1-\frac{(4-x)^3}{20} = 0.75$	M1*		
			$(4-x)^3 = 5$	*M1	Valid attempt to solve. Must produce soln.	$Q^3-12Q^2+48Q-59=0$
			2.29	A1		
				[3]		